Name: Date: Period: 4

Volume and Surface Area of Solids (Pyramids and Cones)

Learning Objective SWBAT derive and apply formulas for volume and surface area of pyramids & cones.

Volume

Watch the YouTube video on the projector. (also posted on Edline, right side of home page)

What does the person say about the similarities between the cone and the cylinder?

bases are $\cong$; same height

The person in the video will now fill up the cylinder using full pours from the cone. How many water-filled cones do you think it will take to fill the cylinder? 3

How many full cones did it take to fill the cylinder? 3

So...

$3(\text{vol. pyramid}) = \text{vol. prism}$

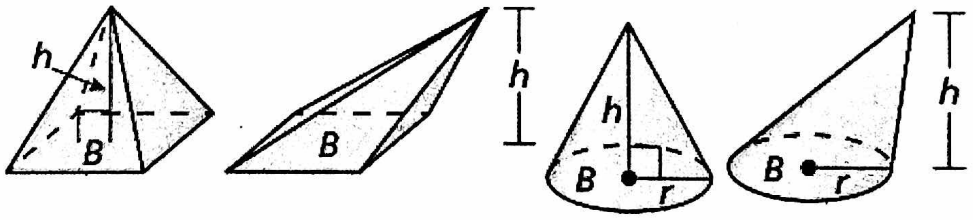
$$\text{vol. pyramid} = \frac{\text{vol. prism}}{3} = \frac{Bh}{3}$$

Volume of a Pyramid or Cone

The volume V of a pyramid or cone is

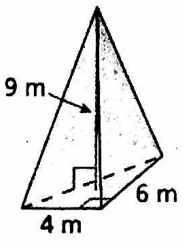
$$\frac{Bh}{3}$$

where B is the area of the base and h is the height.



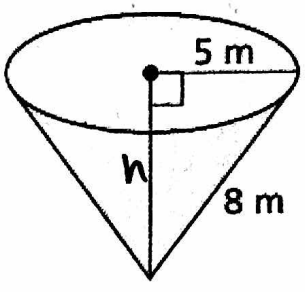
Examples:

1. Find the volume of the pyramid.



$$V = \frac{Bh}{3} = \frac{(4 \cdot 6)(9)}{3} = \boxed{36\ m^3}$$

2. Find the volume of the cone.



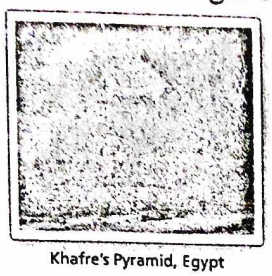
$$V = \frac{Bh}{3} = \frac{\pi 5^2 \cdot \sqrt{39}}{3}$$

$$\boxed{\approx 163.49\ m^3}$$

$h^2 + 5^2 = 8^2$

$h = \sqrt{39}$

3. Originally, Khafre's Pyramid had a height of about 144 meters and a volume of about 2,218,800 cubic meters. Find the side length of the square base.



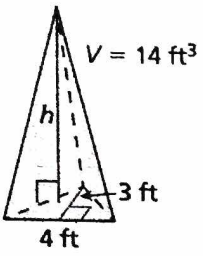
Khafre's Pyramid, Egypt

$2{,}218{,}800 = \frac{144 \cdot B}{3}$

$B = 46225 = s^2$

$s = 215$ **215 m**

4. Find the height of the triangular pyramid



$14 = \frac{h \cdot \frac{1}{2}(3)(4)}{3}$

h = 7

5. Cone A and cone B are similar. Find the volume of cone B.

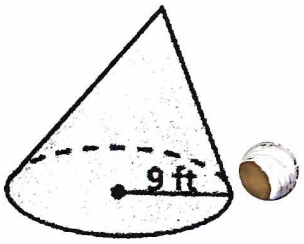
$\left(\frac{3}{39}\right)^3 = \frac{15\pi}{B}$

$\frac{1}{27} = \frac{15\pi}{B} \rightarrow$ **$B = 405\pi$ ft³**

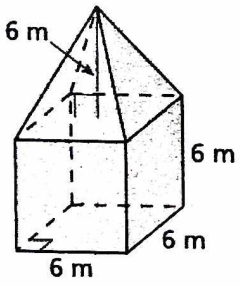
Cone A

$V = 15\pi$ ft³

Cone B



6. Find the volume of the solid.

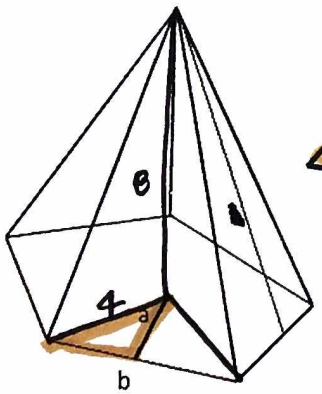


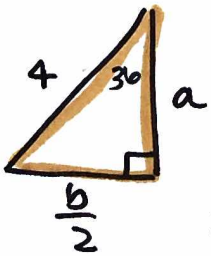
B prism: $(6 \cdot 6)(6) = 216$

pyr: $\frac{(6 \cdot 6)(6)}{3} = 72$

vol $= 216 + 72 =$ **288 m³**

7. Find the volume of a regular pentagonal prism with a height of 8 cm and whose base has a radius of 4 cm.





$\cos 36 = \frac{a}{4}$

$a \approx 3.2\#361$

$\sin 36 = \frac{(\frac{b}{2})}{4}$

$2.3511 = \frac{b}{2}$

$b \approx 4.7023$

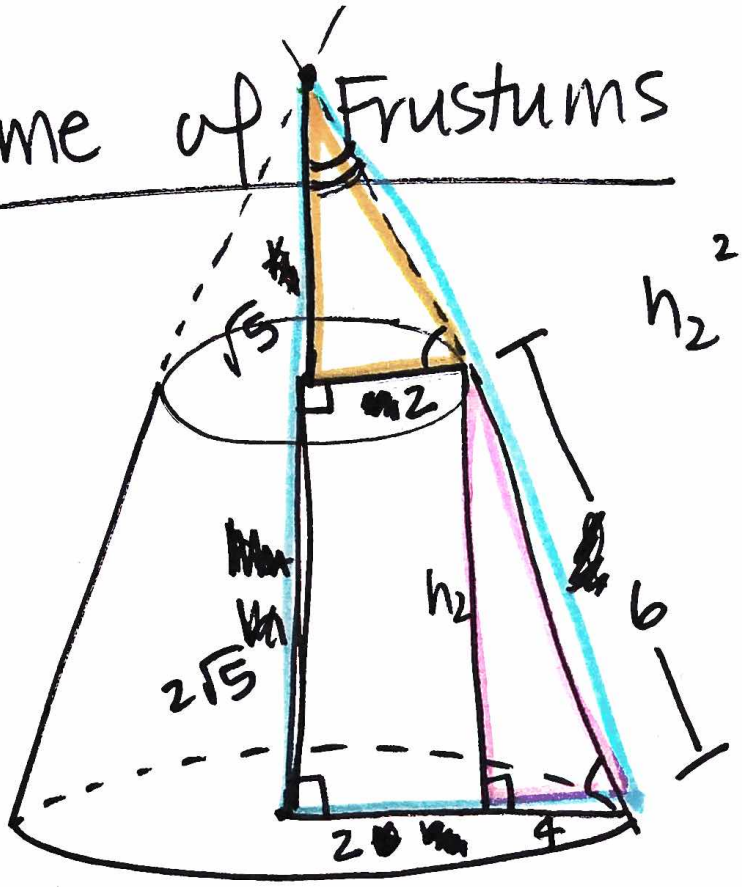
$B = \frac{1}{2}(3.2361)(5)(4.7023)$

≈ 38.0428

$V = \frac{(38.0428)(8)}{3}$

≈ 101.45 cm³

Volume of Frustums



$h_2^2 + 4^2 = 6^2$

$h_2 = 2\sqrt{5}$

$h_1 = \sqrt{5}$

$\Delta \sim$ by AA$\sim$

$r_1 = 2$, $r_2 = 6$ $\quad \ell = 6$

$$V_{frustum} = V_{big\ cone} - V_{small\ cone}$$

$$= \frac{(\pi \cdot 6^2)(3\sqrt{5})}{3} - \frac{(\pi \cdot 2^2)(\sqrt{5})}{3}$$

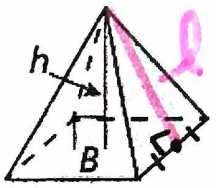
$$= \frac{108\sqrt{5}\pi - 4\sqrt{5}\pi}{3}$$

$$= \frac{104\sqrt{5}\pi}{3} \approx 243.53$$

rface Area

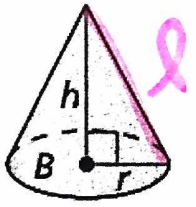
Slant Height:

Pyramids: the distance from the vertex to the midpoint of an edge of the base.
Cones: the distance from the vertex to the circumference of the base.



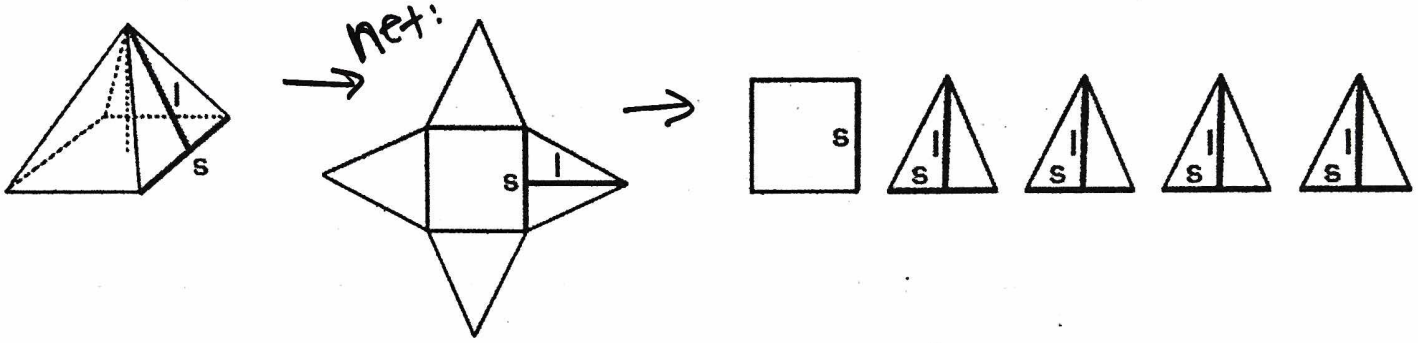
Regular/Right Pyramids and Cones:

Pyramid: the base is a regular polygon, the lateral faces are congruent, height's endpoints are the vertex of the pyramid and the center of the base (polygon).
Cone: height's endpoints are the vertex of the cone and the center of the base (circle)



Deriving the Formula

Consider a square pyramid, and consider its net. Find the sum of the area of all of the smaller figures



$$\text{Lateral area} = \underbrace{\tfrac{1}{2}s\cdot l + \tfrac{1}{2}sl + \cdots + \tfrac{1}{2}sl}_{n \text{ times}}$$

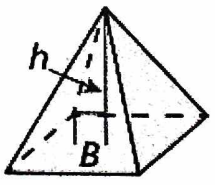
$$= \tfrac{1}{2}s\cdot l\cdot n = \tfrac{1}{2}(s\cdot n)\cdot l = \tfrac{1}{2}Pl$$

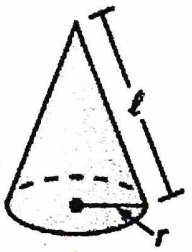
Surface Area of Regular/Right Pyramids and Cones

The surface area S of a regular/right pyramid or cone is

$$SA = \tfrac{1}{2}Pl + B$$

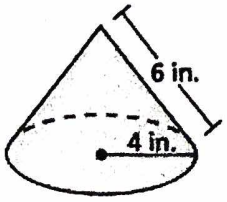
Where B is the area of the base, P is the perimeter of the base, and l is the slant height





Examples:

8. Find the surface area of the right cone.



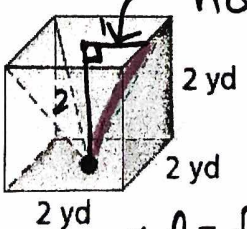
$\frac{1}{2}Pl + B$

$\frac{1}{2}(2\pi \cdot 4)(6) + \pi \cdot 4^2$

$24\pi + 16\pi$

$40\pi \text{ in}^2$

9. Find the surface area of the solid.



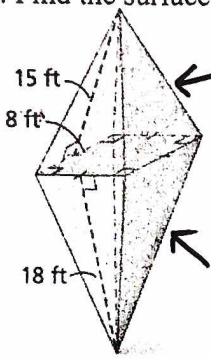
Cube: $(2\cdot 2)(5) = 20$

pyr: $\frac{1}{2}(2\cdot 4)(\sqrt{5})$

$20 + 4\sqrt{5} \approx 28.94 \text{ yd}^2$

$* l = \sqrt{5}$

10. Find the surface area of the solid.

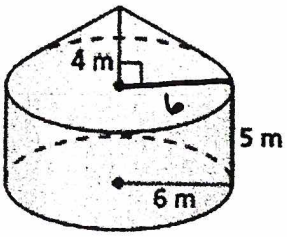


$\frac{1}{2}Pl = \frac{1}{2}(8 \cdot 4)(15) = 240$

$\frac{1}{2}Pl = \frac{1}{2}(8 \cdot 4)(18) = 288$

$SA = 528 \text{ ft}^2$

11. Find the surface area of the solid.

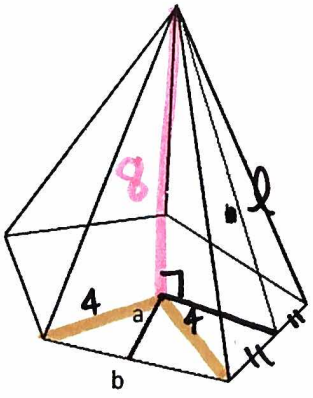


cyl: $Ph + B = (\pi \cdot 2 \cdot 6)(5) + \pi \cdot 6^2 \approx 301.59$

& cone: $\frac{1}{2}Pl = \frac{1}{2}(2 \cdot \pi \cdot 6)(2\sqrt{13}) \approx 135.93$

$SA \approx 437.52 \text{ m}^2$

12. Find the surface area of a regular pentagonal prism with a height of 8 cm and whose base has a radius of 4 cm

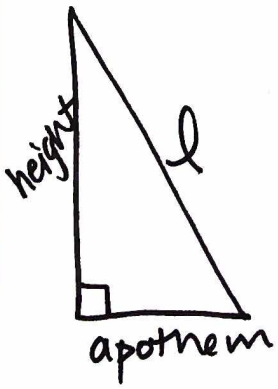


$a \approx 3.2361$
$b \approx 4.7023$
} from ex. #7

$B \approx 38.0428$

$(3.2361)^2 + 8^2 = l^2$

$l \approx 8.6397$

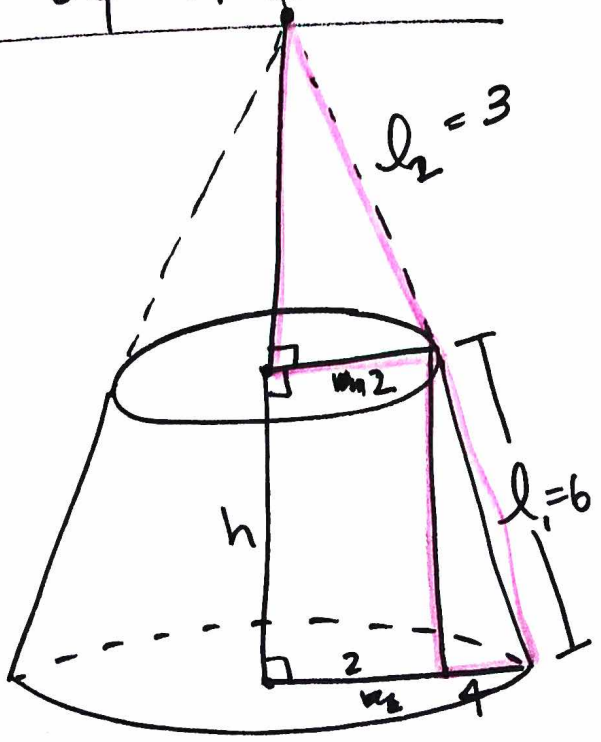


$\frac{1}{2}Pl + B$

$\frac{1}{2}[(4.7023)(5)](8.6397) + 38.0428$

$SA \approx 139.61 \text{ cm}^2$

SA of Frustums



$r_1 = 2$

$r_2 = 6$

$l_1 = 6$

$h = 2\sqrt{5}$

$l_2 = 3$

$\Delta s \sim$ by AA$\sim$

$$LA_{Frustum} = LA_{big\ cone} - LA_{small\ cone}$$

$$= \frac{1}{2}Pl - \frac{1}{2}Pl$$

$$= \frac{1}{2}(2 \cdot \pi \cdot 6)(9) - \frac{1}{2}(2 \cdot \pi \cdot 2)(3)$$

$$= 54\pi - 6\pi$$

$$= 48\pi$$

$+$ 2 bases

$$48\pi + (\pi \cdot 2^2) + (\pi \cdot 6^2)$$

$$48\pi + 4\pi + 36\pi = \boxed{88\pi}$$